AP Physics C **Name: ____**
2022 Summer Assignment
Form A (For those who have completed a calculus course)
DUE: At the beginning of the first day of class

*"… We do these things **not** because they are easy, **but because they are hard**; because **that goal** will serve to organize and measure the best of our energies and skills, because **that challenge** is one that we are willing to accept, one we are unwilling to postpone, and **one we intend to win**!"*

~ JFK, Address at Rice University on the Nation's Space Effort, 1962

Welcome to AP Physics C! AP Physics C is equivalent to the first (and second, if you're staying for E&M) semester college level physics course for physical science and engineering majors. It covers topics in mechanics (and electricity and magnetism, if you're going for the full thang), using calculus methods and concepts to analyze physical situations. Many of these topics you have already encountered freshman year in Concepts of Physics.

AP Physics C requires an exceptional proficiency in algebra, trigonometry, and geometry. A good handle on the calculus concepts encountered in an AB calculus-type course is required as well. In addition to the science concepts, AP Physics C may at times seem like a course in applied mathematics.

It is strongly recommended by the College Board that AP Physics C be taken as a second year physics course (which we do satisfy at the Mount because you've taken Concepts of Physics). The following assignment includes some mathematical problems that are considered routine in AP Physics C, as well as some concepts from freshman year that you need to review over the summer to be ready to hit the ground running in September. Part of the purpose of this summer assignment is to fight your "summer vacation inertia" and get you on the right path to success in the fall. ☺

The attached pages contain a brief review, hints, and example problems. It is hoped that combined with your previous math and physics knowledge, this assignment is merely a review and a means to brush up before school begins in the fall. Please read the text and instructions throughout. If you use any extra paper for scratch work, please attach it to the packet before you turn it in. You are welcome to contact me at any time over the summer if you want to ask a question about anything. This assignment is equal to a major test grade in quarter 1.

It is important that you do this work yourself so you know the material. Looking up answers on the internet or copying someone else's work will do you no good because you will not have practiced the material. While it's fine to ask for help or a hint on a problem here or there, you need to do the bulk of the work yourself to really understand the material.

Please ensure your full name is on every sheet of everything submitted if you're doing it on paper. iPad work is welcome as well.

You will need your AP textbook to do some of this assignment. Let's talk about the textbook situation one-on-one.

Have a GREAT summer. See you in the fall!

Section 1: Math Review (Algebra)

Some of the important skills from algebra that are used on a routine basis in AP Physics include:

- Isolating a variable on one side of an equation (this is the single most important skill)
- The quadratic formula
- Factoring the difference of squares
- Solving simultaneous sets of equations
- Direct proportionality and inverse proportionality (and their corresponding graphs)
- Graphing linear equations

Complete the following exercises. Show all your work.

Solve each equation for the variable indicated.

1. $d = \frac{1}{2}at^2$; Solve for t.

2. $T = 2\pi\sqrt{\frac{L}{g}}$; Solve for L.

3. $g = \frac{GM}{d^2}$; Solve for d.

6. $a = \frac{v_f - v_0}{t}$; Solve for v_f.

7. $U = mgh$; Solve for h.

8. $v = \sqrt{2gy}$; Solve for y.

4. $x - x_0 = v_0 t + \frac{1}{2} a t^2$; Solve for t.

9. $\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{tot}}$; Solve for k_{tot}.

5. $\ln\left(\frac{v}{v_0}\right) = \frac{bt}{m}$; Solve for v.

10. $\frac{x-y}{6} = \frac{x+y}{4} - 1$; Solve for y.

Solve the following sets of equations as indicated.

13. Eliminate T and write an equation for a in terms of the other variables.

$T - f - m_1 g \sin\theta = m_1 a$ $\qquad$ $m_2 g - T = m_2 a$

14. Solve for T by eliminating v.

$v = \frac{2\pi r}{T}$ $\qquad$ $\frac{GM}{r^2} = \frac{v^2}{r}$

15. Show that the following two equations can be reduced to the single equation:

$$x + y = a + b$$

$x - y = a - b$

$(x^2 - y^2) = (a^2 - b^2)$

16. Consider the equation $F = \frac{Gm_1m_2}{d^2}$.

a) Sketch a graph of F as a function of m_1 for positive values of F and m_1.

b) Sketch a graph of F as a function of d for positive values of F and d.

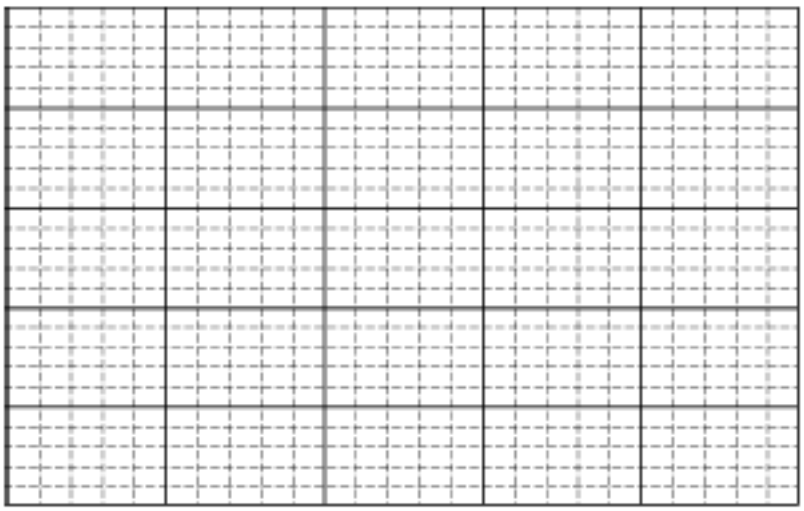
c) Sketch a graph of F as a function of d^2 for positive values of F and d^2.

Making and interpreting graphs using data.

Note that the convention in physics will ALWAYS be to plot y vs. x. This means that if you are asked to plot a graph of velocity vs. time, you know immediately that velocity will be plotted on the y-axis and time will be plotted on the x-axis.

17. In the space provided, plot a graph of velocity vs. time, using the data given. Be sure to label all axes, including units.

Time (s)	1	2	3	4	5	6
Velocity (m/s)	3.6	7.5	11.0	14.9	18.4	22.3



a) What kind of curve did you obtain?

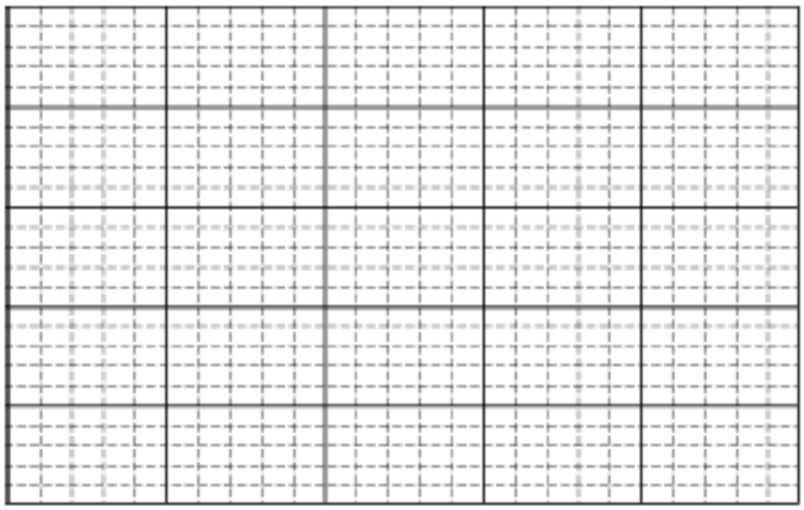
b) What is the relationship between the variables?

c) What was the velocity at $t = 5.5$ s?

d) What would you expect the velocity to be at $t = 10$ s?

18. In the space provided, plot a graph of frequency vs. wavelength, using the data given. Be sure to label all axes, including units.

Wavelength (nm)	300	400	500	600	700	800
Frequency (Hz)	10^{15}	7.5×10^{14}	6×10^{14}	5×10^{14}	4.3×10^{14}	3.75×10^{14}



a) What kind of curve did you obtain?

b) What is the relationship between the variables?

c) What would be the frequency when the wavelength is 350 nm?

d) What would be the wavelength when the frequency is 6.5×10^{14} Hz?

Graphs are VERY important in physics because they show patterns between variables. A straight line graph that starts from the (0,0) point is the best proof that two variables are directly proportional. You should remember from Algebra 1 that the general equation for a straight line is $y = \mathrm{m}x + \mathrm{b}$, where m is the slope of the graph and b is the y-intercept.

Other types of graphs that you need to remember are quadratic graphs ($y = \mathrm{a}x^2 + \mathrm{b}x + \mathrm{c}$), inverse graphs ($y = 1/x$), inverse square graphs ($y = 1/x^2$), square root graphs ($y = \sqrt{x}$), exponential graphs (more on that in precalc review), and sine/cosine graphs. These are the ones frequently seen in AP Physics C.

Although the above list is important, when it comes to finding a relationship between two variables the only graph that can show this very clearly is the straight line graph.

<u>Example</u>

Let's say that you want to prove the relationship between the kinetic energy of an object and its speed. You plot speed on the x-axis and kinetic energy on the y-axis. You will get a curve which as you know is a parabola (since the kinetic energy is directly proportional to the square of the speed).

Now let's say you do another experiment that, unknown to you, also follows the same pattern. You will also get a curve when you plot the graph. Will you be able to recognize that this is a parabola? What if it is a curve that is very close to a parabola but not quite?

What can you do to be sure that you have cracked the relationship?

Think again about the example above. If instead of plotting kinetic energy against speed you plot kinetic energy against speed squared what will you get? You will get a straight line through the origin! Moreover, you will be certain that the relationship is that the kinetic energy is directly proportional to the speed squared.

So what have we learned so far?

ALWAYS AIM AT PLOTTING TWO VARIABLES THAT WILL GIVE YOU A STRAIGHT LINE!

Try these exercises:

19. The equation for the period of a pendulum, as we will prove, is $T = 2\pi\sqrt{\frac{L}{g}}$.
    a) Suppose you perform an experiment where you measure L and then correspondingly measure the effect on T. What variables should you plot against each other in order to get a graph that is a straight line?

20. The universal gravitational law, as we'll see, is given by the equation $F = \frac{Gm_1m_2}{r^2}$.

a) What variables should you plot against each other in order to prove that the force F is directly proportional to the product of the masses of the objects?

b) What variables should you plot against each other in order to prove that the force is inversely proportional to the distance squared (r^2) between the objects?

<u>The significance of the slope</u>

Throughout this course you will be asked to decide which graphs to plot in order to show a relationship or to calculate a physical constant.

We have already noted how important it is to aim at plotting a graph that will end up being a straight line. This gives you a definite answer about the relationship between the two variables. But there is more to it. The slope of this line will give you information about a constant in your experiment.

<u>Example</u>

Let's say that you want to measure the gravitational field strength of Earth with a pendulum. You vary the length and measure the period. You then decide to plot T^2 against L. The graph will be a straight line. What will its slope be? To find this, compare the pendulum equation with the straight line equation as shown below:

$$T^2 = 4\pi^2 \frac{L}{g}$$
$$y = mx + b$$

You should be able to see that T^2 corresponds to y, L corresponds to x. $4\pi^2/g$ corresponds to m, and 0 corresponds to b. We will be doing this often in lab experiments so it's important that you get comfortable with the process.

Apart from its use as explained above, the slope in all curves (straight lines or otherwise) corresponds to the **derivative** of the function you plot (more on this in the differential calculus section). This is why if you plot time on the x-axis and displacement on the y-axis the slope corresponds to the velocity of the object. If the line is curved the slope does not stay the same, which means that it is equal to the instantaneous velocity of the object.

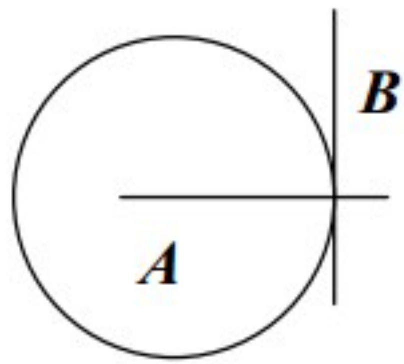
Section 2: Math Review (Geometry/Trig)

Some of the important skills from geometry and trigonometry that are used on a routine basis in AP Physics include:

- Complementary angles
- Relationship between tangent lines and radial lines for circles
- Alternate interior angles of a line
- Right triangle trigonometry
- Pythagorean theorem (very frequently)
- The sine, cosine, and tangent functions
- Inverse sine, cosine and tangent functions
- Drawing a normal line

Solve the following geometric problems.

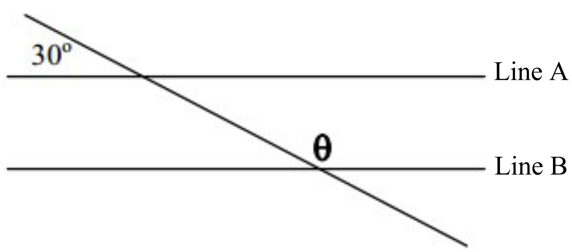
1. Line **B** touches the circle at a single point. Line **A** extends through the center of the circle.



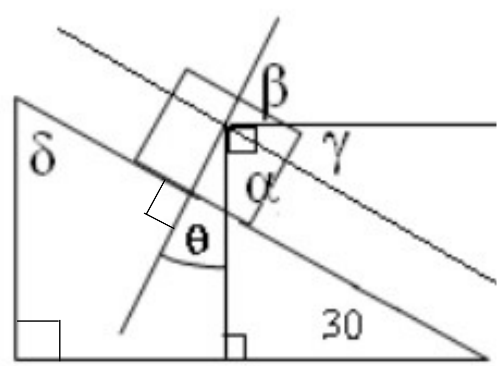
   a) Line **A** is called a ________________ line and line **B** is called a _________________ line.

   b) What must be the angle between lines A and B?

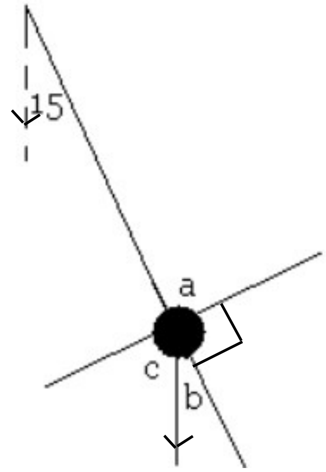
2. What is angle θ? Assume Line A || Line B.



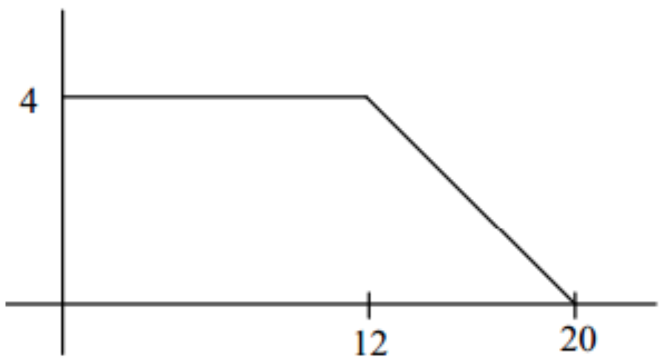
3. How large are angles α, β, γ, δ, and θ?



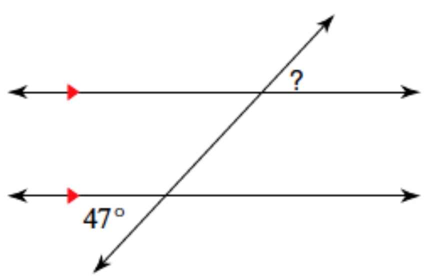
4. How large are angles *a, b*, and *c*?



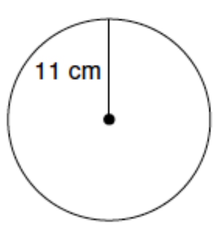
5. What is the area under the graph?



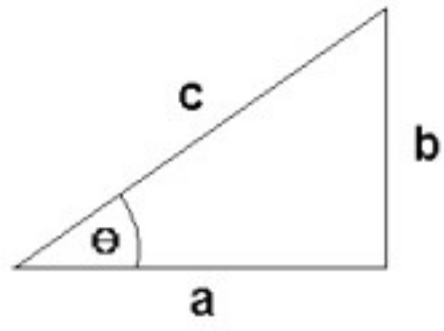
6. What is the angle in question?



7. Determine the circumference (in meters) and the area (in square meters) of the circle below.



Use the generic triangle shown below, right-triangle trigonometry, and the Pythagorean theorem to solve the following problems. Note that your calculator must be set to degree mode.



8. $\theta = 55^{\circ}$ and **c** = 32 m; solve for **a** and **b**.

9. $\theta = 45^{\circ}$ and **a** = 15 m/s; solve for **b** and **c**.

10. **b** = 17.8 m and $\theta = 65^{\circ}$; solve for **a** and **c**.

11. **a** = 250 m and **b** = 180 m; solve for θ and **c**.

12. **a** = 25 cm and **c** = 32 cm; solve for **b** and θ.

Section 3: Math Review (Precalculus + Vectors)

Some of the important skills from precalculus that are used in AP Physics include:

- Graphs of the sine and cosine functions
- Some common trig identities
- Exponential growth and decay and their corresponding graphs
- Radian measure
- Vectors

1. For the function $x = A\sin(\omega t + \phi)$,
   a) Which variable represents the amplitude?

   b) What is the period of the function?

   c) Which variable represents the phase shift?

2. Consider the function, $x = 2\sin(\frac{\pi}{2}t)$, where x is measured in meters and t is measured in seconds. (The units on $\pi/2$ are s^{-1}.)
   a) What is the amplitude of the function?

   b) What is the period of the function?

   c) Sketch a graph of the function, showing two full periods.

   d) Where is the particle at $t = 0$?

3. Consider the function, $x = -1.5\cos(2\pi t)$, where x is measured in meters and t is measured in seconds. (The units on 2π are s^{-1}.)
   a) What is the amplitude of the function?

   b) What is the period of the function?

   c) Sketch a graph of the function, showing two full periods.

   d) Where is the particle at $t = 0$?

4. Consider the function $v = v_0(1 - e^{-\frac{bt}{m}})$.
   a) Sketch a graph of the function.

   b) As t increases indefinitely, what value does v approach?

Trig Identities

The most common trig identities used in AP Physics are below. Complete the identity.

1. $\sin^2\theta + \cos^2\theta =$

2. $\frac{\sin\theta}{\cos\theta} =$

3. $\sin(2\theta) =$

4. $\cos(90 - \theta) =$

5. $\sin(90 - \theta) =$

You will be expected to recognize these when they are encountered.

Convert the following angle measurements in degrees to radians. Leave your answer in terms of π.

5. 30°
6. 45°
7. 60°
8. 90°
9. 180°
10. 270°
11. 360°

Vectors

Many of the important quantities in physics are vectors. This makes proficiency in vectors extremely important. A vector is a physical quantity with both a magnitude and a direction. (Examples: velocity, acceleration, force)

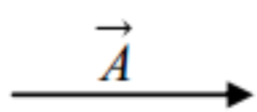
A <u>scalar</u> is a physical quantity described by a single number with units. (Examples: time, mass, and temperature)

<u>Magnitude</u> refers to size or extent. It is the numerical value of the physical quantity represented by the vector.

<u>Direction</u> is the alignment or orientation of the vector with respect to a coordinate system.

<u>Notation</u>
In your textbook, vector quantities are always indicated by bold type, like this: **A**
In your notes simply put an arrow over the letter representing the vector, like this $\vec{A}$

Vectors are drawn as arrows:



The <u>length</u> of the arrow is proportional to the vector's magnitude.
The <u>direction</u> the arrow points is the direction of the vector.

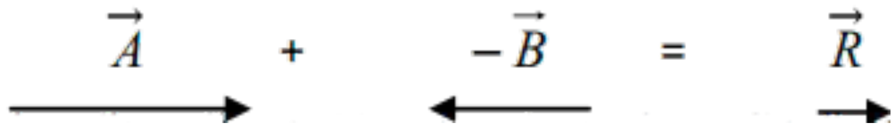
<u>Vector Addition and Subtraction</u>
Think of it as vector addition only. The result of adding vectors is called the resultant **R**.

$\vec{A}+\vec{B}=\vec{R}$

If **A** has a magnitude of 3 and **B** has a magnitude of 2, then **R** has a magnitude of 3 + 2 = 5.

When you need to subtract one vector from another, think of the one being subtracted as being a "negative" vector (its direction being 180º from the original vector). Then add them:

A – **B** = **R** is really **A** + **-B** = **R**

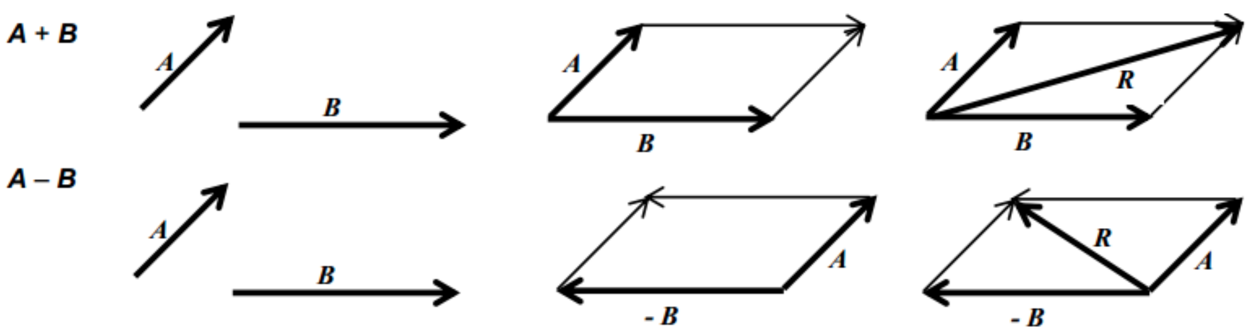


A "negative" vector has the same length as its positive counterpart, but its direction is reversed. If **A** has a magnitude of 3 and **B** has a magnitude of 2, then **R** has a magnitude of 3 + (-2) = 1.

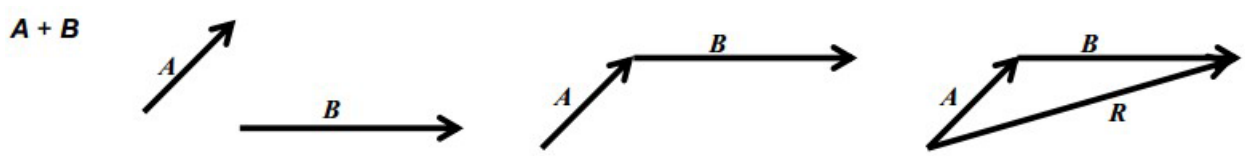
<u>This is VERY important</u>: In physics a negative number does not always mean a smaller number. In math class, -2 is smaller than +2, but in physics these numbers have the exact same magnitude, they just point in different directions (180º apart).

There are two methods of adding vectors.

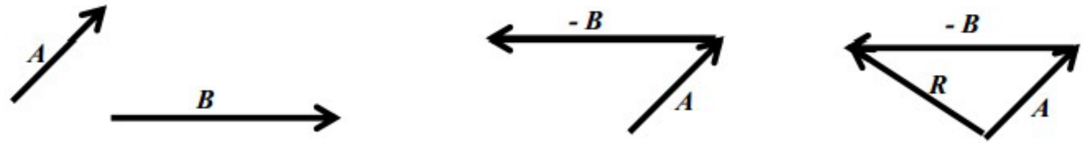
<u>Parallelogram Method</u>



<u>Tip-to-Tail Method</u>

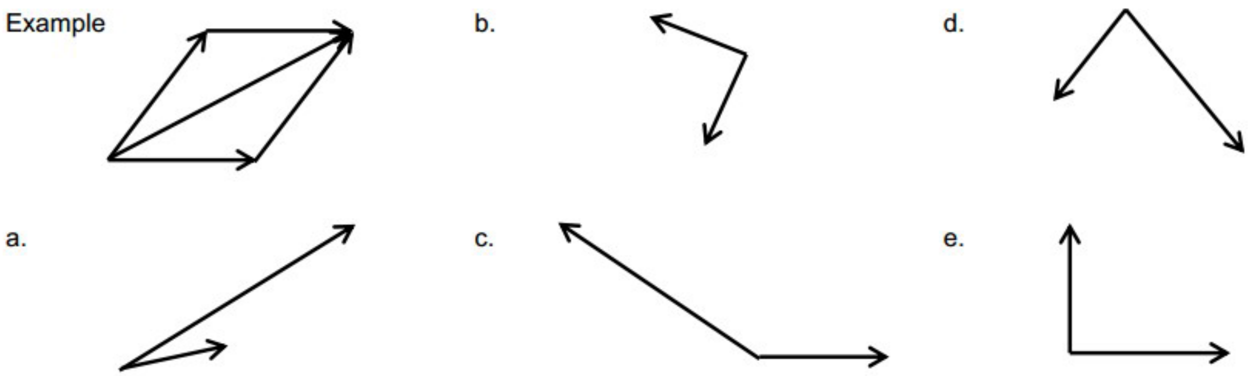


A – B

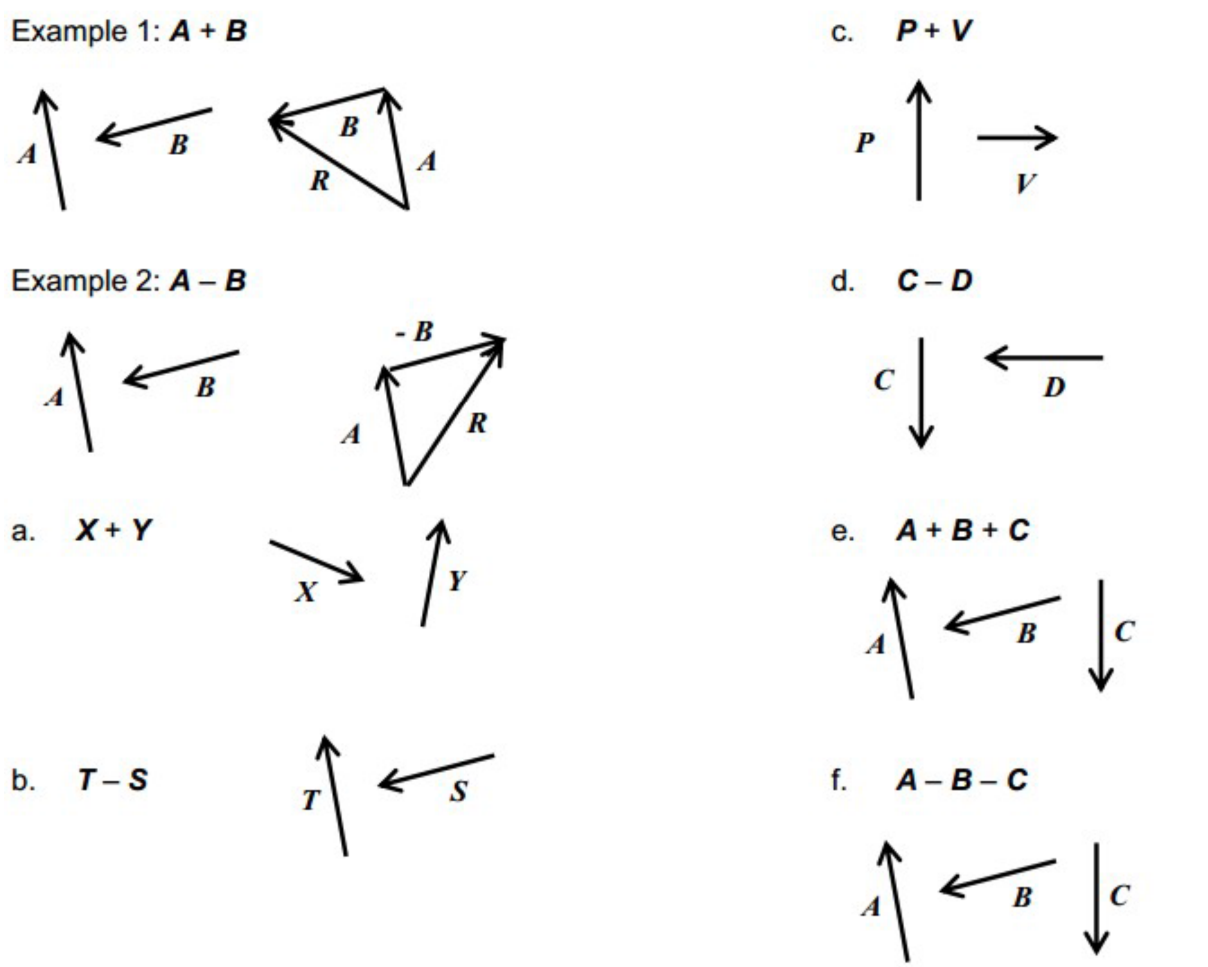


It should be readily apparent that both methods arrive at the exact same solution since each method is essentially a parallelogram. It is useful to understand both systems. In some problems one method is advantageous, while in other problems the alternative method is superior.

12. Draw the resultant vector using the parallelogram method of vector addition.

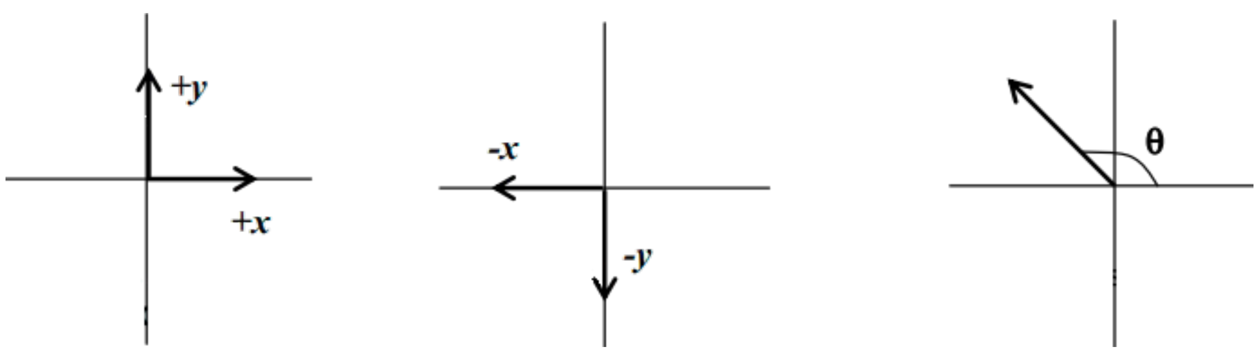


13. Draw the resultant vector using the tip to tail method of vector addition. Label the resultant as vector **R**.



<u>Direction</u>

What does positive or negative direction mean? How is it referenced? The answer is the coordinate axis system. In physics a coordinate axis system is used to give a problem a frame of reference. Positive direction is a vector moving in the positive x, positive y, or positive z direction, while a "negative" vector moves in the negative x, negative y, or negative z direction.

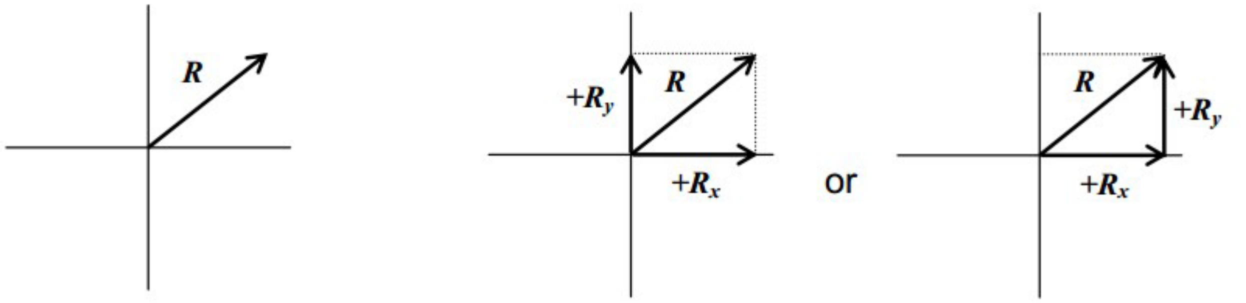


What about vectors that don't fall on the axis? You must specify their direction using degrees measured from the +x direction.

Component Vectors

A resultant vector is a vector resulting from the sum of two or more other vectors. The resultant has the same magnitude and direction as the total of the vectors that compose the resultant. Could a vector be described by two or more other vectors? Would they have the same total result?

This is the reverse of finding the resultant. You are given the resultant and must find the component vectors on the coordinate axis that describe the resultant.

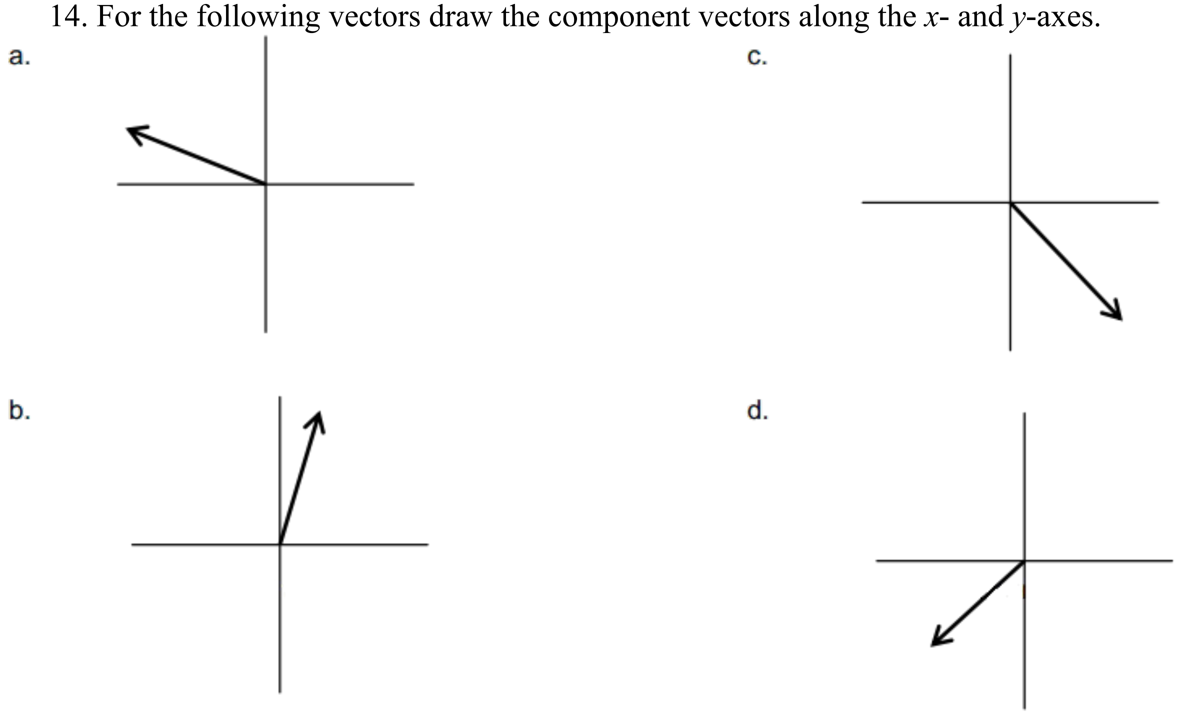


Any vector can be described by an x-axis vector and a y-axis vector which, when summed together, have a resultant that equals the original vector. The advantage is you can then use plus and minus signs for direction instead of the angle.

You should also watch the following two videos:

• http://www.khanacademy.org/science/physics/v/introduction-to-vectors-and-scalars
• http://www.khanacademy.org/science/physics/v/visualizing-vectors-in-2-dimensions

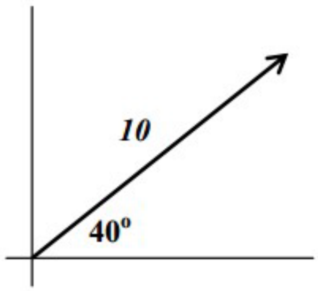
14. For the following vectors draw the component vectors along the x- and y-axes.

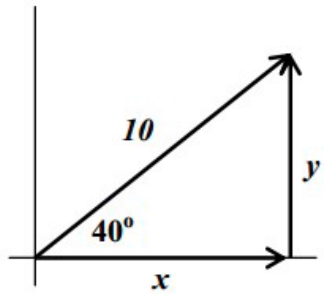


Obviously the quadrant that a vector is in determines the sign of the x- and y-component vectors.

Trigonometry and Vectors

Given a vector, you can now draw the *x*- and *y*-component vectors. The sum of vectors *x* and *y* describe the vector exactly. Again, any math done with the component vectors will be as valid as with the original vector. The advantage is that math on the *x*- and/or *y*-axis is greatly simplified since direction can be specified with plus and minus signs instead of degrees. But, how do you mathematically find the length of the component vectors? Use trigonometry.





$$\cos\theta = \frac{adj}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$x = hyp\cos\theta \qquad y = hyp\sin\theta$$

$$x = 10\cos 40^o \qquad y = 10\sin 40^o$$

$$x = 7.66 \qquad y = 6.43$$

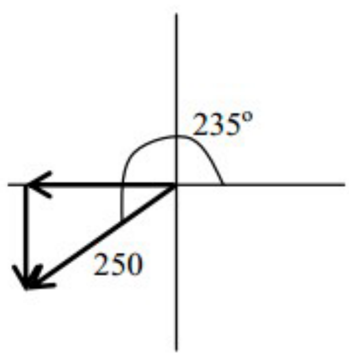
Solve the following problems. You will be converting from a polar vector, where direction is specified in degrees measured counterclockwise from east, to component vectors along the x- and y-axes. Remember the plus and minus signs on you answers. They correspond with the quadrant the original vector is in.

Draw the vector first to help you see the quadrant. Anticipate the sign on the *x*- and *y*- components. Do not bother to change the angle to less than 90º, using the value given will result in the correct ± signs.

The first number given will be the vector's magnitude, and the second the degrees from east. You calculator must be in degree mode.

Example

250 at 235º



$$x = hyp\cos\theta$$

$$x = 250\cos 235^o$$

$$x = -143$$

$$y = hyp\sin\theta$$

$$y = 250\sin 235^o$$

$$y = -205$$

15. 89 m at 150º

16. 6.50 cm at 345º

17. 0.00556 m/s at 60º

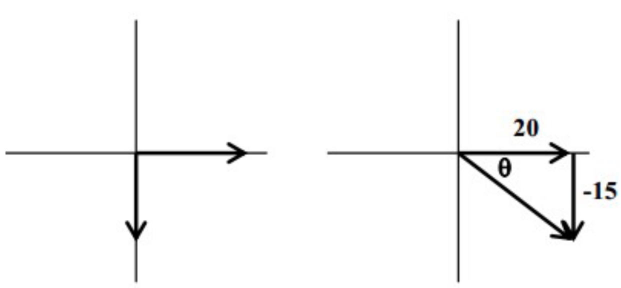
18. 750 N at 180º

19. 12 km/h at 265º

20. 990 cm at 320º

Given two component vectors solve for the resultant vector. This is the opposite of the previous set of problems. Use the Pythagorean theorem to find the hypotenuse, then use inverse tangent to solve for the angle.

Example: ***x*** = 20, ***y*** = -15



$$R^2 = x^2 + y^2 \qquad \tan\theta = \frac{opp}{adj}$$

$$R = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{opp}{adj}\right)$$

$$R = \sqrt{20^2 + 15^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = 25$$

$$360^o - 36.9^o = 323.1^o$$

21. **x** = 600 m, **y** = 400 m

22. **x** = – 0.75 m. **y** = – 1.25 m

23. **x** = – 32 m/s, **y** = 16 m/s

24. **x** = 0.0065 cm, **y** = – 0.0090 cm

25. **x** = 20,000 m/s, **y** = 14,000 m/s

26. **x** = 325 cm, **y** = 998 cm

Section 4: Math Review (Limits & Differential Calculus)

Some of the important skills from differential calculus that are used in AP Physics include:

- Understanding the concept of a limit as an interval approaches zero
- Taking the derivative of a polynomial function
- Taking the derivative of sine and cosine functions
- The concept of derivative as the slope of a curve
- The concept of derivative as the rate of change of a function
- Finding the maximum or minimum of a function using derivatives

You should already understand a little about limits from your honors precalc class. In this part of the assignment, you will review limit concepts, then extend that into basic differential calculus.

<u>Limits</u>

Recall that the slope m of a graph of f at the point $(x, f(x))$ is equal to the slope of its tangent line at $(x, f(x))$, and is given by

$$m = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

1. Use this limit approach to find the slope of the following.

   a) $f(x) = x^2$ at the point $(-2, 4)$

   b) $f(x) = -2x + 4$

Recall that the **derivative** of f at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

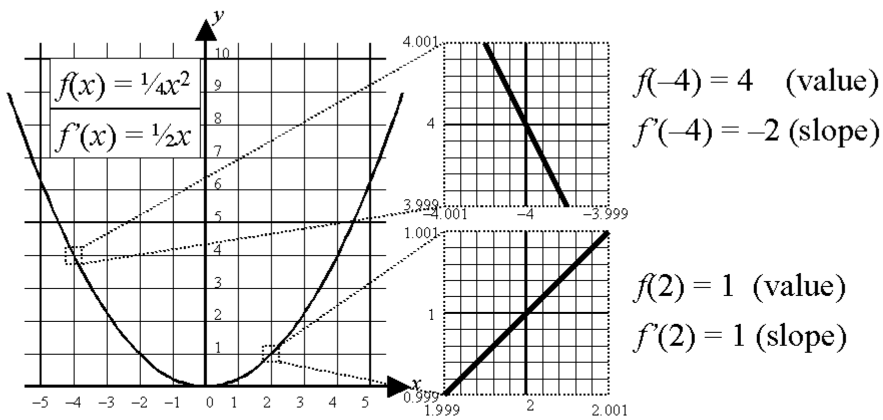
2. Use this approach to find the derivative of the following.
   a) $f(x) = 3x^2 - 2x$

   b) $f(x) = \sqrt{x}$

Review of Derivatives

"____________________" is a mathematical operation that acts on a function $y = f(x)$. The result is called the "____________________" of f, and it is also a function, $\dot{y} = f'(x)$.

The derivative gives the __________ of the function $f(x)$ at a given value of x. It also represents the "___________________________________" of a function $f(x)$ at a particular value x.

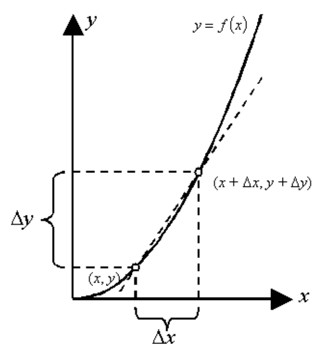


It takes __________ points to establish a line and calculate a slope. To find the slope at a single point (x, y) on a function y = f(x), we choose another point (x + Δx, y + Δy) on the graph.

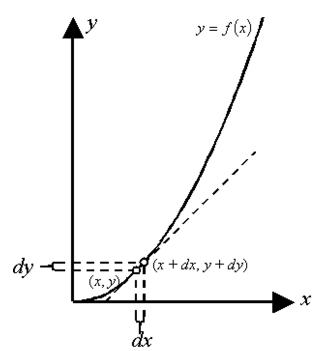
Constant Rule

If $f(x) = c$ (a constant), then

$f'(x) =$__________________



$$\text{slope} \approx \frac{\Delta y}{\Delta x}$$



$$\text{slope} = \frac{dy}{dx}$$

Power Rule

If $f(x) = cx^n$ (where c and n are constants and $n \neq 0$), then $\frac{df}{dx} =$ _________________________.

Addition/Subtraction Rule

If $y = f(x) \pm g(x)$ then $\frac{dy}{dx} =$ _________________________.

Constant Multiple Rule

If $y = cf(x)$ (where c is a constant), then $\frac{dy}{dx} =$ _________________________.

Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] =$	$\frac{d}{dx}[\cos x] =$	$\frac{d}{dx}[\tan x] =$
$\frac{d}{dx}[\sec x] =$	$\frac{d}{dx}[\csc x] =$	$\frac{d}{dx}[\cot x] =$

Derivatives of Exponential and Logarithmic Functions

$\frac{d}{dx}[\ln x] =$	$\frac{d}{dx}[e^x] =$

Find the derivative of each of the following functions.

3. $f(x) = 4x^3$

   $\frac{df}{dx} =$

4. $x = 2t^5 + 4t^2 + 2t$

   $\frac{dx}{dt} =$

5. $f(w) = \sqrt{w} + 4\sqrt[3]{w} - \frac{2}{w} + \frac{1}{3w^2}$
$\frac{df}{dw} =$

6. $f(x) = 6\sin x - 2\cos x$
$\frac{df}{dx} =$

Product and Quotient Rule

Let $f(x)$ be the product of two different functions $u(x)$ and $v(x)$. In other words, $f(x) = u(x)v(x)$. Then the derivative of f is found using the functions u and v and their derivatives:

$$\frac{df}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Now consider the situation where $f(x)$ is the quotient of two functions $u(x)$ and $v(x)$. In other words, $f(x) = \frac{u(x)}{v(x)}$. Then the derivative of f is given by:

$$\frac{df}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Chain Rule

Let y be a function of f, which is itself a function of g. In other words, $y = f(g(x))$. Then the derivative of y is:

$$\frac{dy}{dx} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Find the derivative of the following.

7. $y = (\sin x)(\ln x)$
$\frac{dy}{dx} =$

8. $a = v^2 \cos v$

$\frac{da}{dv} =$

9. $y = \frac{\sin x}{\cos x}$

$\frac{dy}{dx} =$

10. $k = \sin(m^2)$

$\frac{dk}{dm} =$

11. $k = (\sin m)^2$

$\frac{dk}{dm} =$

12. $v = \sin(\ln t)$

$\frac{dv}{dt} =$

13. $f(x) = (4x^2 + 2x)^3$

$\frac{df}{dx} =$

14. $f(x) = \frac{4-x^3}{x^2+1}$
$\frac{df}{dx} =$

15. $z = 2\cos(2x+3)$
$\frac{dz}{dx} =$

16. $x = 0.5\sin(\frac{\pi}{2}t + \frac{\pi}{3})$
$\frac{dx}{dt} =$

17. $x = 4\ln(3y)$
$\frac{dx}{dy} =$

Second Derivative

The second derivative of a function $f(x)$, noted _____ or __________, is the ________________ __________________________________.

Example: Find the second derivative of $y = e^x \sin x$

$$\frac{dy}{dx} = e^x \cos x + (\sin x)\, e^x$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = e^x(-\sin x) + (\cos x)e^x + (\sin x)e^x + e^x(\cos x)$$

$$= 2e^x \cos x$$

Find the second derivative of each of the functions below.

18. $x = -3t^3 - 2t + 4$

$\frac{d^2x}{dt^2} =$

19. $z = 2x^5 + 4x^2 + 2x$

$\frac{d^2z}{dx^2} =$

20. $v = 2x^4 + 2x^2 + 2x$

$\frac{d^2v}{dx^2} =$

21. $f(x) = \sin(x^2)$

$\frac{d^2f}{dx^2} =$

22. $s = (4w^2 + 2w)^3$

$\frac{d^2s}{dw^2} =$

23. $f(x) = \frac{4-x^3}{x^2+1}$

$\frac{d^2f}{dx^2} =$

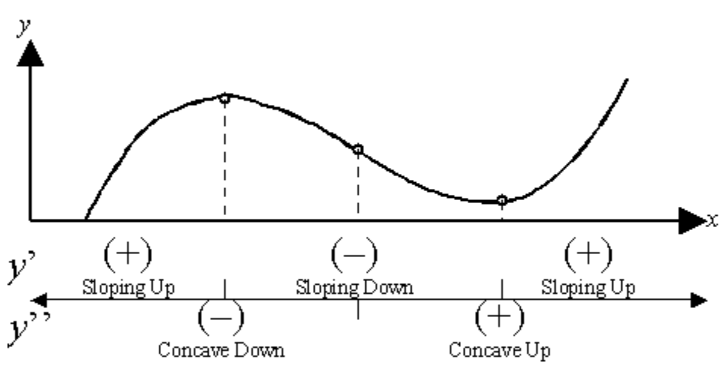
24. $f(x) = 2\cos(2x+3)$

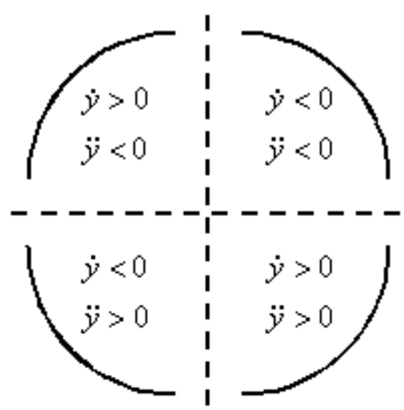
$\frac{d^2f}{dx^2} =$

25. $x = 0.5\sin(\frac{\pi}{2}t + \frac{\pi}{3})$

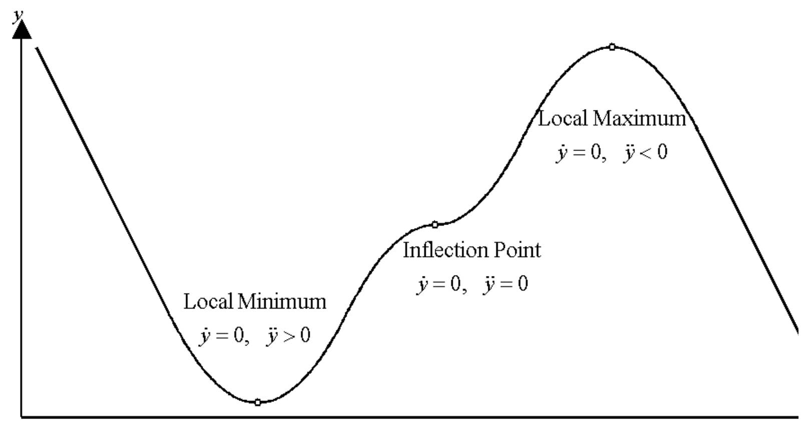
$\frac{d^2x}{dt^2} =$

Finding Extrema (Maximums and Minimums)





Critical Point – Value of x where ______. A critical point can be one of three things:



Local Maximum – Point where the value of $f(x)$ is

____

____

Local Minimum – Point where the value of $f(x)$ is

____

____

Inflection Point – Point where the function

____

____

___

A critical point can also be where $f'(x)$ is undefined, but we will not focus on this in physics.

For the following, find the local minima and maxima of the functions and determine which are minima and which are maxima.

26. $f(x) = x^3 - 3x^2 + 7x - 5$

27. $f(x) = (2x^2 + 1)^2$

Section 5: Math Review (Integral calculus)

Some of the important skills from integral calculus that are used in AP Physics include:

- Taking the integral of a polynomial function
- Recognizing when an integral solves as natural log
- Taking the integral of sine and cosine functions
- The concept of integral as area under a curve
- Setting up and solving a differential equation

1. Explain in your own words the meaning of the definite integral of a function.

2. Explain in your own words the meaning of an anti-derivative of a function.

3. Explain how derivatives and integrals are related to each other by the fundamental theorem of calculus. Your explanation does not have to be overly precise, just tell me what the basic relationship is.

4. $f(x) = x^2 + 3x + 2$

$\int f(x)dx =$

$\int_0^3 f(x)dx =$

5. $f(x) = -3x^3 - 2x + 4$

$\int_{-1}^{1} f(x)dx =$

6. $f(x) = 6x^5 + 3x^2 + 2x$

$\int f(x)dx =$

$\int_0^1 f(x)dx =$

7. $f(x) = x^4 + 4x^3 + 2x - 4$

$\int_{-4}^{0} f(x)dx =$

8. $f(x) = -3\sin x$

$\int_{\pi}^{2\pi} f(x)dx =$

9. $f(\theta) = 3\cos\theta + 2\sin\theta + \theta^2$

$\int_{\pi}^{2\pi} f(\theta)d\theta =$

$\int_{-\pi}^{\pi} f(\theta)d\theta =$

10. $f(x) = \frac{1}{x+3}$

$\int_{0}^{4} f(x)dx =$

11. $f(t) = e^{-\frac{bt}{m}}$

$\int f(t)dt =$

Determine the area under the curve for the following functions.

12. $f(x) = 5x^2$, [0, 2]

13. $f(x) = x^2 - 2x - 4$; [– 2, 4]

Section 6: Unit Conversions & The Metric System

Physics uses the International System of Units (abbreviated SI from French: Le Système International d'Unités) with the MKS base unit system. MKS stands for meter, kilogram, and second. These are the units of choice of physics, and most other units are derived from these three. The equations in physics depend on unit agreement so you must convert to MKS in most problems to arrive at the correct answer. Common conversions encountered in AP Physics C are:

- kilometers (km) to meters (m) and meters to kilometers
- centimeters (cm) to meters (m) and meters to centimeters
- millimeters (mm) to meters (m) and meters to millimeters
- micrometers (μm) to meters (m) and meters to micrometers
- nanometers (nm) to meters (m) and meters to nanometers
- grams (g) to kilograms (kg) and kilograms to grams

Other conversions will be taught as they become necessary.

It is expected that you be skilled at unit conversions and that you know the most common metric prefixes by heart (kilo, centi, milli).

Convert the following. Show all work.

1. 4008 g to kg
2. 1.2 km to m
3. 823 nm to m
6. 25.0 μm to m
7. 2.65 mm to m
8. 8.23 μm to m

4. 0.77 m to cm

9. 40.0 cm to m

5. 8.8×10^{-8} m to mm

10. 1.5×10^{11} m to km

A very useful method of converting one unit to an equivalent unit is called the factor-label method of unit conversion. Perhaps you used this in chemistry. Suppose you are given the speed of an object as 25 km/h and wish to express it in m/s. To make this conversion, you must change km to m and h to s by multiplying by a series of factors so that the units you do not want will cancel out and the units you want will remain.

Conversion: 1000 m = 1 km and 3600 s = 1 h

$$25\,\frac{\cancel{km}}{\cancel{h}} \times \frac{1000\,m}{1\,\cancel{km}} \times \frac{1\,\cancel{h}}{3600\,s} \approx 6.94\,\frac{m}{s}$$

Carry out the following conversions using the factor-label method. Show all your work.

11. How many seconds are in a year?

12. How many centimeters are in 24 kilometers?

13. How many milligrams are in 55 kilograms?

14. How many kilometers per month are in 3×10^8 m/s?

15. How many cm^2 are in 18.8 m^2?

16. What is the volume of a sphere, in m^3, if its radius is 6.7 cm?

Read chapter 1 in your textbook and summarize the standard units of measurements.

Length: The Meter

Time: The Second

Mass: The Kilogram

NOTE: The definition of the kilogram is in the process of changing (actually methinks it's officially changed by now)! Google it. Write what you find here, too.

Section 7: Scientific Notation & Order of Magnitude

It is expected that you be comfortable working with large or small numbers expressed in scientific notation. It is also expected that you understand how to perform calculations with scientific notation accurately using your calculator.

Practice with the following exercises. Place the answer in scientific notation when appropriate and simplify the units. Scientific notation is used when it takes less time to write than the ordinary number does. As an example 200 is easier to write than 2.00 x 10^2, but 2.00 x 10^8 is easier to write than 200,000,000. Cancel units, and show the simplified units in the final answer.

1. $F = \frac{6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}(2\times10^{30}kg)(6\times10^{24}kg)}{(1.5\times10^{11}m)^2}$ $\qquad$ $F =$

2. $g = \frac{6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}(6\times10^{24}kg)}{(6.38\times10^{6}m)^2}$ $\qquad$ $g =$

3. $E = (5 \times 10^{-3}kg)(3 \times 10^8\,\frac{m}{s})^2$ $\qquad$ $E =$

4. $K = \frac{1}{2}(3.6 \times 10^{-4}kg)(4.7 \times 10^5\,\frac{m}{s})^2$ $\qquad$ $K =$

5. $p = (9.8 \times 10^5 kg)(1.5 \times 10^{-6} \frac{m}{s})$ $p =$

6. $v_f = 10^4 \frac{m}{s} - 10^3 \frac{m}{s}$ $v_f =$

7. $\varphi = \frac{9.92\times10^9 \frac{m}{s^{-1}}}{2.43\times10^{-12} \frac{m}{s^{-1}}}$ $\varphi =$

Order of magnitude refers to the approximate measure of the size of a number, equal to the $\log_{10}$ rounded to a whole number, using the usual rules for rounding.

Usual rules for rounding:

If the number you are rounding is followed by 5, 6, 7, 8, or 9, round the number up. Example: 3.8 rounded to the nearest whole number is 4. If the number you are rounding is followed by 0, 1, 2, 3, or 4, round the number down. Example: 3.4 rounded to the nearest whole number is 3.

A number such as 3.45 rounded to the nearest whole number would be 3, because 3.45 is closer to 3 than it is to 4.

As an example of order of magnitude, the order of magnitude of 30,000 is 4, because 30,000 = 3 $\times$ 10^4. Watch a classic video on orders of magnitude: https://www.youtube.com/watch?v=0fKBhvDjuy0.
If that link doesn't work, google "Powers of Ten video 1977".

An order of magnitude calculation involves estimating an answer based on which power of 10 it is closest to.

Example 1: The mass of a proton is 1.67 x 10^{-27} kg. Order of magnitude, this is ~10^{-27} kg.

Example 2: The acceleration due to gravity at Earth's surface is about 9.8 m/s^2. Order of magnitude, this is ~10m/s^2. Using 10 instead of 9.8 makes calculations easier and quicker.

Give the order of magnitude of the following. You will have to look up some values. Show all your work.

8. The mass of a hydrogen-1 atom, in kg.

9. The mass of Earth plus the mass of the Sun, in kg.

10. 10 years plus one day, expressed in seconds.

11. The mass of the Moon plus the mass of the Earth, in kg.

12. 300 kilometers plus 10 meters.

Section 8: Newton's Laws & Other Things You Need To Remember From Freshman Year

So as to be able to spend our class time on the deeper ideas and problems, it is expected that you are comfortable with the statements of Newton's three laws of motion and that you can easily answer simple questions involving them.

State Newton's three laws of motion in your own words. Explain any terms that need to be explained.

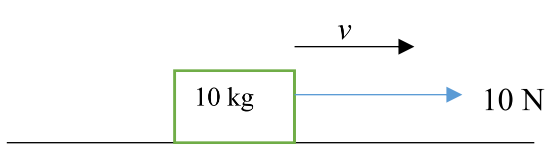
Newton's 1^{st} Law:

Newton's 2^{nd} Law:

Newton's 3^{rd} Law:

Newton's Laws Exercises.

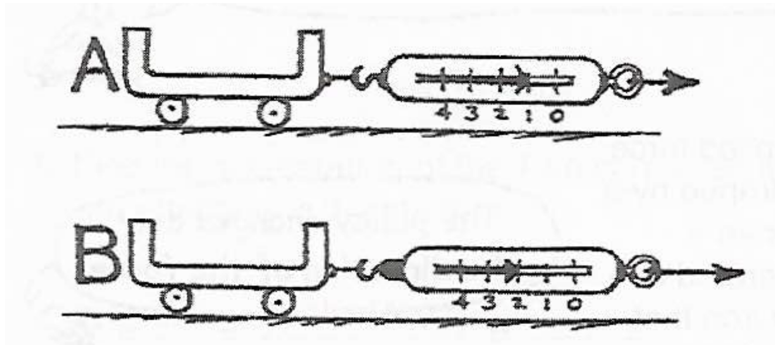
1. The 10 kg box shown in the figure below is sliding to the right along the floor. A horizontal force of 10 N is being applied to the right. The force of friction between the box and the floor is 2 N.



   a) What is the net force on the box?

   b) Is the box in equilibrium?

2. Consider the carts shown below. Cart A has a mass of 1 kg and is pulled with a force of 1 N. Cart B also has a mass of 1 kg but is pulled with a force of 2 N. Which undergoes the greater acceleration?



3. Carolyn exerts a horizontal force *F* on a book of mass *m* on a horizontal surface where the force of friction is *f*.
   a) Write an equation for the resulting acceleration.

   b) Calculate the acceleration when her push is 10 N, the mass of the book is 0.5 kg, and the friction force is 5 N.

4. A car, traveling with a velocity of 30 m/s to the right, slows to a stop in 5 s. The net force on the car is 6000 N.
   a) What is the acceleration of the car? Give magnitude and direction.

   b) What is the car's mass?

Concepts of Physics Review Questions

5. Categorize the following as a "vector" or a "scalar": acceleration, velocity, work, force, speed, distance, mass, displacement, kinetic energy

6. A ball is thrown straight up. It then returns to the same height it started.
   a) What is the direction of the velocity on the way **up**? Is the magnitude of the velocity increasing, decreasing or constant?

   b) What is the direction of the velocity on the way **down**? Is the magnitude of the velocity increasing, decreasing or constant?

   c) What is the direction of the velocity at the highest point? What is the magnitude of the velocity?

   d) What is the direction of the acceleration on the way **up**? Is the magnitude of the acceleration increasing, decreasing or constant?

   e) What is the direction of the acceleration on the way **down**? Is the magnitude of the acceleration increasing, decreasing or constant?

   f) What is the direction of the acceleration at the highest point? What is the magnitude of the acceleration?

7. For the following, is the object speeding up, slowing down or moving at a constant speed? An object has
   a) A positive acceleration and a positive velocity.

   b) A positive acceleration and a negative velocity.

   c) A negative acceleration and a positive velocity.

   d) A negative acceleration and a negative velocity.

   e) A zero acceleration and a positive velocity.

   f) A zero acceleration and a negative velocity.

8. Compare the velocity and acceleration of an object slowing down.

9. A ball is thrown upward with an initial velocity v_o. The ball reaches height h in time t. What is the acceleration of the ball at the highest point?

10. An object has a weight of 550 N. Calculate the object's mass.

Congratulations, you're done!
See you in August!